

## NONLINEAR BULGING FACTOR BASED ON R-CURVE DATA

David Y. Jeong and Pin Tong<sup>1</sup>  
U.S. Department of Transportation  
Research and Special Programs Administration  
Volpe National Transportation Systems Center  
Cambridge, MA 02142-1093 USA

523-29  
23117  
p. 12

## SUMMARY

In this paper, a nonlinear bulging factor is derived using a strain energy approach combined with dimensional analysis. The functional form of the bulging factor contains an empirical constant that is determined using R-curve data from unstiffened flat and curved panel tests. The determination of this empirical constant is based on the assumption that the R-curve is the same for both flat and curved panels.

## INTRODUCTION

Bulging refers to the rotation and deflection of the edges of a longitudinal crack in a pressurized fuselage. Physically, the bulging phenomenon causes local bending at the crack tips which increases the effective stress intensity factor. The conventional engineering approach to account for this effect is to apply a bulging factor to the stress intensity factor.

Bulging factors have been developed both analytically and empirically. For example, one of the most commonly used bulging factors is an empirical formula derived by Kuhn [1]:

$$\beta_B = 1 + 10 \left( \frac{a}{R} \right), \quad \frac{R}{t} > 100 \quad (1)$$

---

<sup>1</sup> Also, Hong Kong University of Science and Technology, Kowloon, Hong Kong.

The obvious shortcoming of this bulging factor is the apparent independence of bulging on skin thickness. Analytical bulging factors have been derived by Folias [2,3] and by Erdogan and Kibler [4]. For example, Folias [3] derived the following bulging factor

$$\beta_B = \sqrt{1 + 0.317\lambda^2} \quad , \quad \lambda = \frac{\alpha}{\sqrt{Rt}} \sqrt[4]{12(1 - \nu^2)} \quad (2)$$

where  $\alpha$  is the half crack length,  $E$  is the modulus of elasticity,  $\nu$  is Poisson's ratio,  $R$  is the radius of curvature, and  $t$  is the skin thickness. Folias [3] states that equation (2) is applicable for all values of  $\lambda$ . However, the analytical bulging factors [2-4] tend to overestimate the bulging effect in most practical cases. This conclusion has been supported by research performed by Ansell [5] who showed that more realistic deformation behavior in the vicinity of the crack tip can be achieved using a geometrically nonlinear analysis rather than a linear analysis. The nonlinear character of crack bulging has been taken into account by Chen [6] who derived a bulging factor that depends on the applied stress. Furthermore, Broek [7] has noted that bulging causes membrane tension which, in turn, produces the main resistance to bulge formation rather than bending stiffness. Consequently, the bulging factor becomes nonlinear because the membrane stress depends on the depth of the bulge.

In this paper, a nonlinear bulging factor is derived using a strain energy approach similar to that used by Chen [6], but combined with dimensional analysis. The resulting bulging formula contains an empirical constant which is determined using R-curve data from unstiffened flat and curved panel tests. The numerical value of this empirical constant is found by assuming that the R-curve or crack resistance curve is the same for both flat and curved panels.

## DERIVATION OF NONLINEAR BULGING FACTOR

A strain energy approach is used to derive the mathematical form of the nonlinear bulging factor. The following derivation initially resembles that used by Chen [6].

The energy release rate is related to the derivative of the strain energy with respect to half-crack length, and can be written as

$$\mathcal{G} = \frac{d}{d\alpha}(F - U) \quad (3)$$

where  $F$  is the work done by external forces and  $U$  is the elastic strain energy of the system associated with crack extension. For a pressurized cylinder, the quantity  $F - U$  is assumed to be comprised of two components; one due to crack bulging and another due to the applied pressure<sup>2</sup>, or

$$\mathcal{G} = \frac{d}{d\alpha}(F_{bulge} - U_{bulge} + F_{flat} - U_{flat}) \quad (4)$$

The component due to the crack face deformation or bulging is derived by assuming that the out-of-plane deformation field in the neighborhood of the crack can be characterized by the pyramidal shape shown schematically in Figure 1. Denoting  $s_1$  and  $s_2$  as characteristic lengths, and  $w_o$  as the maximum out-of-plane displacement, Chen [6] found that

$$\frac{d}{d\alpha}(F - U)_{bulge} \approx \frac{1}{3} p s_1 s_2 \frac{dw_o}{d\alpha} \quad (5)$$

where  $p$  is the applied pressure.

The component due to the applied pressure or hoop stress is

$$\frac{d}{d\alpha}(F - U)_{flat} = p^2 R^2 \frac{\pi \alpha}{E t} = \mathcal{G}_{flat} \quad (6)$$

---

<sup>2</sup> The present derivation uses this decomposition as a simplifying assumption. However, this assumption implicitly decouples the effects of applied loading and bulging. Strictly speaking, these effects should be coupled.

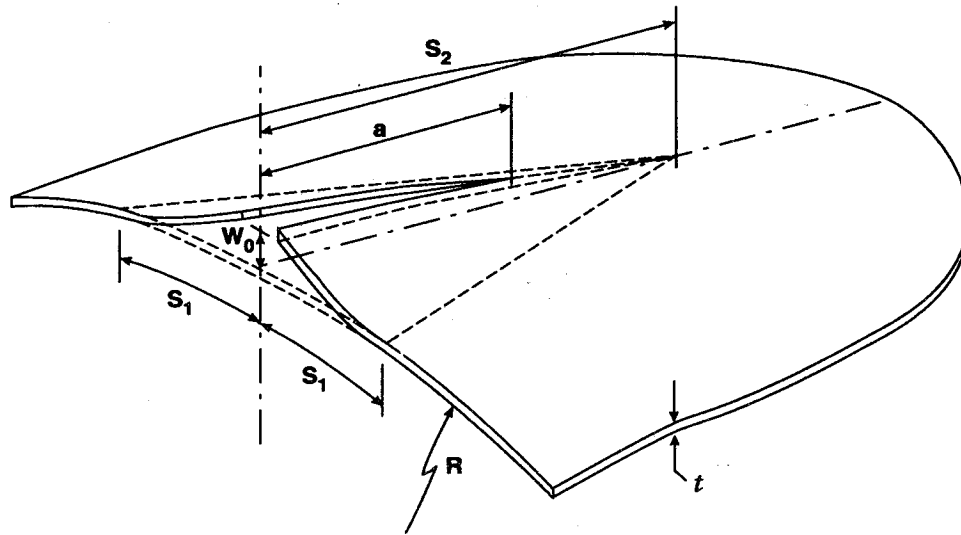


Figure 1. Deformation of crack faces due to bulging.

Combining equations (5) and (6), the energy release rate for a panel with bulging becomes

$$\mathcal{G}_{curve} = \frac{1}{3} p s_1 s_2 \frac{dw_0}{d\alpha} + p^2 R^2 \frac{\pi \alpha}{Et} . \quad (7)$$

The bulging factor is defined as the ratio of stress intensity factors for curved to flat panels, or

$$\beta_B = \frac{K_{I_{curve}}}{K_{I_{flat}}} . \quad (8)$$

The bulging factor is assumed to be related to the energy release rates for panels with and without bulging by

$$\beta_B = \sqrt{\frac{\mathcal{G}_{curve}}{\mathcal{G}_{flat}}} . \quad (9)$$

Thus, the bulging factor has the following mathematical form:

$$\beta_B = \sqrt{1 + \frac{E t s_1 s_2}{3 \pi \alpha p R^2} \left( \frac{dw_o}{d\alpha} \right)} \quad (10)$$

At this point, the derivation of the functional form of the bulging factor deviates from Chen's approach [6], and dimensional analysis is employed. That is, dimensionless constants are used to simplify the mathematical representation of the bulging factor. For example, the characteristic lengths,  $s_1$  and  $s_2$ , can be expressed in terms of a proportion to half crack length,  $\alpha$ ; or

$$s_1 = \alpha_1 \alpha, \quad s_2 = \alpha_2 \alpha \quad (11)$$

In addition, an expression for the out-of-plane deformation, which has its basis in large displacement theory of elasticity [8], can be written as

$$w_o = \alpha_3 \alpha \sqrt[3]{\frac{p \alpha}{E t}} \quad (12)$$

where  $\alpha_3$  is a constant. Therefore, the increment of out-of-plane deformation with respect to crack length can be written as

$$\frac{dw_o}{d\alpha} = \alpha_4 \sqrt[3]{\frac{p \alpha}{E t}} = \alpha_4 \sqrt[3]{\frac{\sigma_o \alpha}{E R}} \quad (13)$$

since the hoop stress is

$$\sigma_o = \frac{p R}{t} \quad (14)$$

Thus, a general nonlinear bulging factor may be found by combining equations (10), (11), (13) and (14):

$$\beta_B = \sqrt{1 + \alpha \left[ \left( \frac{E}{\sigma_o} \right) \left( \frac{\alpha}{R} \right)^2 \right]^{2/3}} \quad (15)$$

where  $\alpha$  is an empirical constant. The determination of the numerical value for this empirical constant is described in the following section.

## DATA ON STABLE TEARING

A series of flat and curved panel tests has been conducted by Foster-Miller, Inc. (FMI) to provide a database from which analytical models can be verified [7,9]. R-curve data were collected during these tests in terms of stable crack extension versus applied pressure. For flat panels, crack resistance in terms of the stress intensity factor can be calculated as

$$K_R = \sigma_o \sqrt{\pi(\alpha_o + \Delta\alpha)} \sqrt{\sec\left(\frac{\pi(\alpha_o + \Delta\alpha)}{W}\right)} \quad (16)$$

where  $W$  is the width of the panel. A regression analysis was performed to fit the FMI flat panel data to the following two-parameter R-curve equation:

$$K_R = K_o \Delta\alpha^b \quad (17)$$

The results of the regression analysis were:  $K_o = 106.1 \text{ ksi-in}^{1/2}$  and  $b = 0.212$ .

For curved panels, crack resistance in terms of stress intensity factor depends on the bulging factor<sup>3</sup>:

$$K_R = \sigma_o \beta_B \sqrt{\pi(\alpha_o + \Delta\alpha)} \sqrt{\sec\left(\frac{\pi(\alpha_o + \Delta\alpha)}{W}\right)} \quad (18)$$

---

<sup>3</sup> This equation is an approximation to the actual stress intensity factor for fracture resistance because the curved test panels are biaxially loaded which has not been taken into account.

In principle, if the assumption is made that the R-curve is the same for both flat and curved panels, then the constant,  $\alpha$ , can be determined by combining equations (17) and (18), and equating the result with the nonlinear bulging factor described by equation (15), or

$$\frac{K_o \Delta \alpha^b}{\sigma_o \sqrt{\pi(\alpha_o + \Delta \alpha)} \sqrt{\sec\left(\frac{\pi(\alpha_o + \Delta \alpha)}{w}\right)}} = \sqrt{1 + \alpha \left[ \left( \frac{E}{\sigma_o} \right) \left( \frac{\alpha_o + \Delta \alpha}{R} \right)^2 \right]^{2/3}} \quad (19)$$

Using this equation, a different numerical value for the empirical constant can be calculated for each collected data point where the amount of stable crack extension was measured at a given level of applied stress. A total of 251 data points were collected during the unstiffened curved panel test. Each point was used to calculate the numerical value of the empirical constant. The average or mean value of  $\alpha$  was found to be 0.671<sup>4</sup>.

Figure 2 compares the flat and curved panel data, as derived by this method, with the regression curve. Good agreement between the test data and the two-parameter R-curve equation is evident between 0 to 0.6 inches of stable crack extension. After  $\Delta \alpha = 0.6$  inches, the curved panel test data are approximately 10% higher than the regression curve in terms of stress intensity factor.

## DISCUSSION

Figure 3 compares the nonlinear bulging factor or equation (15) where  $\alpha = 0.671$  with other bulging factors, namely, equations (1) and (2). The nonlinear bulging factor is approximately equal to equation (1) at extremely small values of crack length. As the crack length increases, however, the nonlinear bulging factor deviates from the Folias bulging factor which is based on the assumptions of linear elastic material behavior and small displacements. Residual strength tests were conducted on curved panels with ratios of half crack length to radius of curvature between 0.06 to 0.10. Figure 3 shows that over the range of test values that the difference between the empirical and the nonlinear bulging factors is not as significant as that between the empirical and Folias solution.

---

<sup>4</sup> For the FMI curved panels,  $E$  was assumed to be 10 msi and  $R$  is 75 inches.

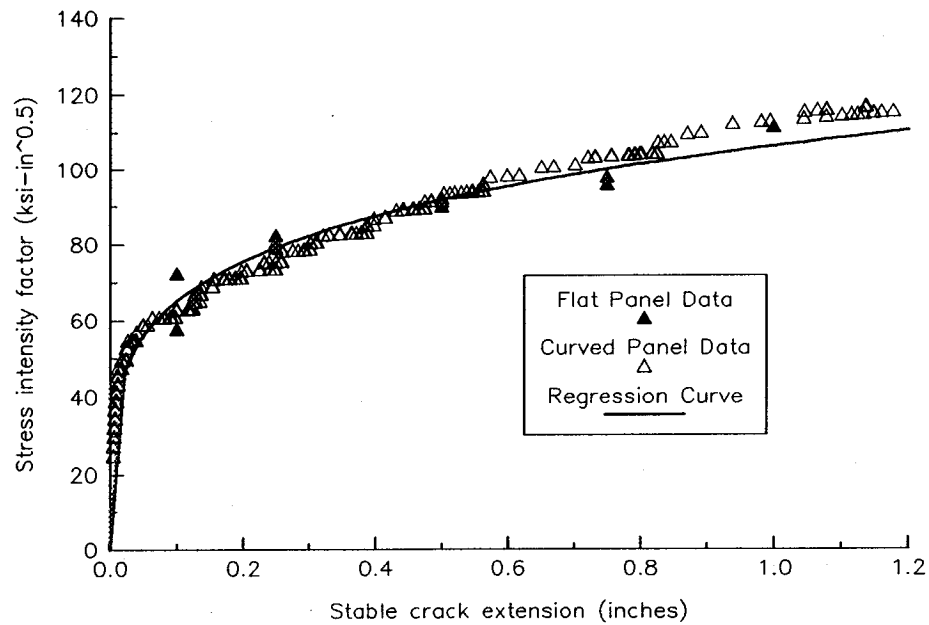


Figure 2. R-Curve for flat and curved panels.

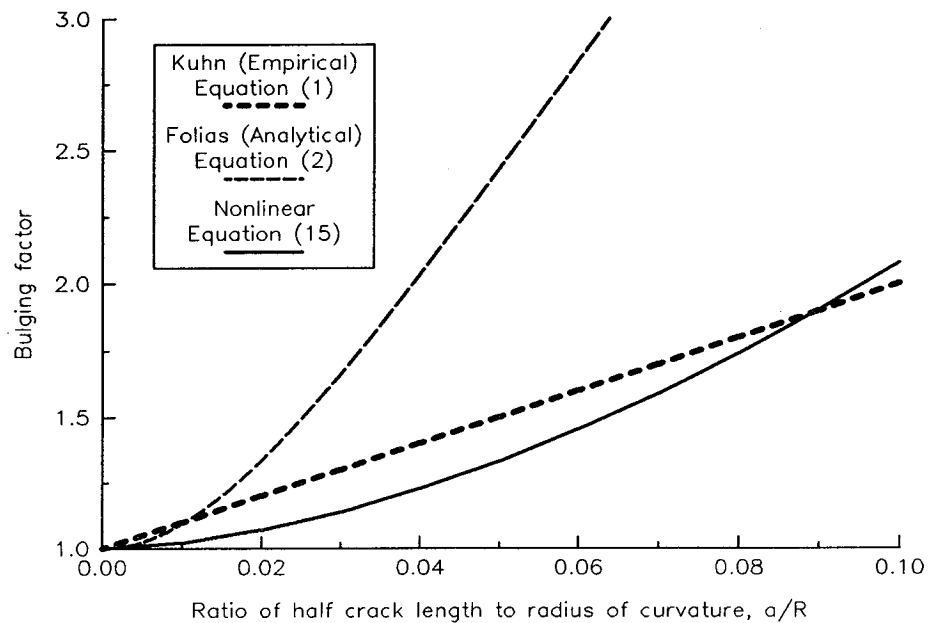
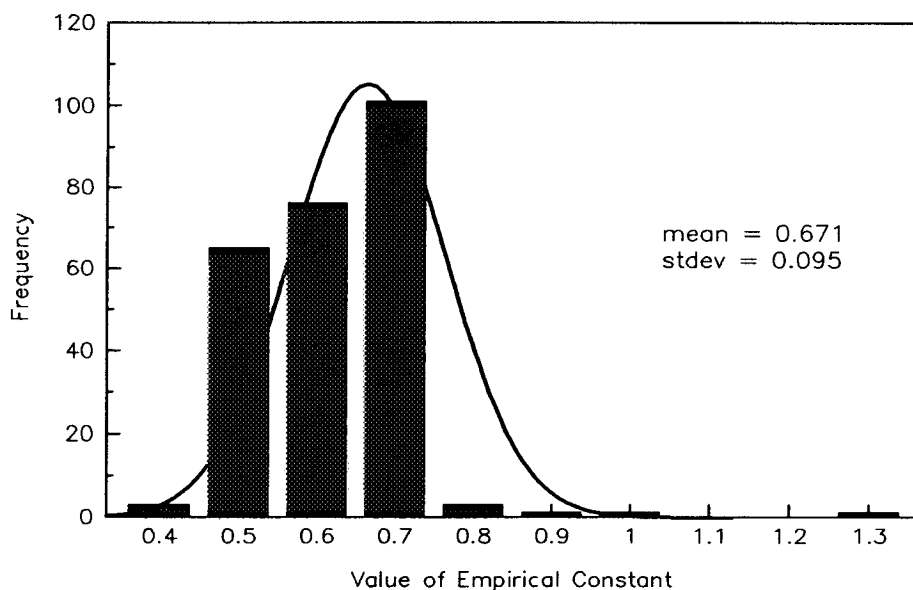


Figure 3. Comparison of bulging factors ( $R = 75$  inches and  $t = 0.040$  inch).



As mentioned previously, the numerical value of the empirical constant was determined by calculating the constant at each data point on the R-curve from the FMI curved panel tests, and taking the average or mean value. Figure 4 shows the frequency distribution of the calculated values for the empirical constant. A normal distribution curve is also shown for comparison. The mean value was found to be 0.671, and the standard deviation was 0.095. The frequency plot indicates that 242 of 251 data points resulted in a value between 0.5 and 0.7. Thus, the mean value of 0.671 appears to be reasonable.

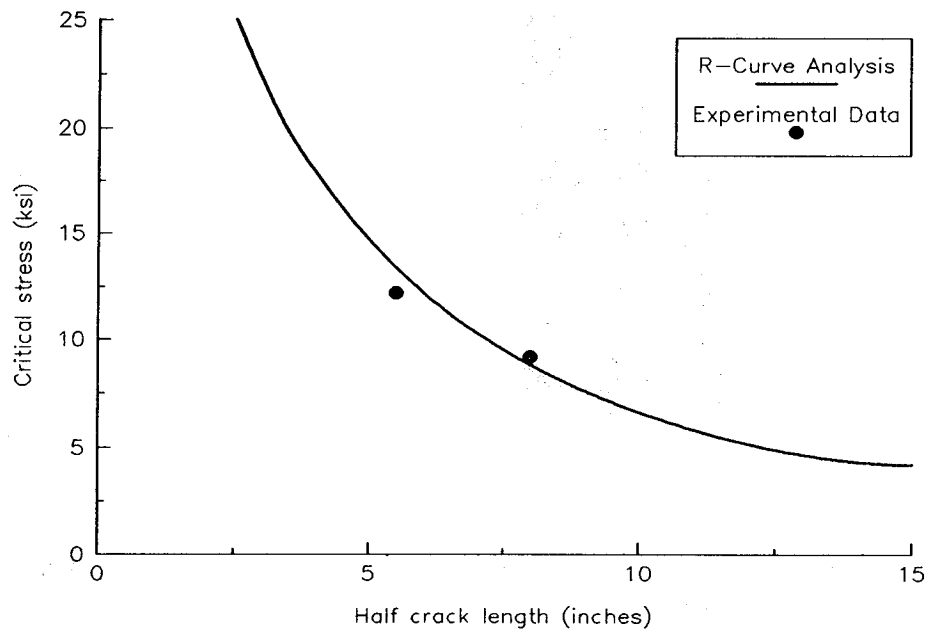


**Figure 4.** *Distribution of numerical values for empirical constant in nonlinear bulging factor.*

Predictions of failure were made for the Foster-Miller curved panels using the two-parameter R-curve equation and the nonlinear bulging factor. Two tests were conducted at the FMI full-scale test facility with unstiffened curved panels [9]. Table 1 lists the results of these tests which used two different values of initial crack length. The table compares the test results with the failure predictions based on the R-curve analysis. Predictions of failure stress are shown to be within 10% of the experimental data, which is within reasonable engineering agreement. Figure 5 shows a plot comparing the R-curve predictions with the experimental results.

**Table 1. Failure Stresses for Foster-Miller Unstiffened Curved Panels**

Initial half-crack length (inches)	Failure stress (ksi)		% difference
	Prediction	Experiment	
5.5	13.4	12.2	+9.8%
8.0	8.8	9.2	-4.3%



**Figure 5. Failure predictions for FMI curved panels using R-curve data and nonlinear bulging factor.**

The effect of biaxial loading has not been explicitly taken into account in the present methodology. Bulging is affected by the ratio of hoop-to-longitudinal stress. Chen [6] found that the bulge factor of a biaxially loaded specimen is significantly lower than that of a uniaxially loaded specimen.

Since the results presented in this paper were generated on the basis of two unstiffened curved panels, the results are considered to be developmental. While additional research is recommended in the determination of appropriate bulging factors, the approach presented in this paper appears to give encouraging results.

## CONCLUSIONS

The mathematical or functional form of a nonlinear bulging is derived using a strain energy approach similar to that used by Chen [6]. The derivation is complemented with dimensional analysis which leads to an unknown or empirical constant in the nonlinear bulging factor.

A methodology to determine the numerical value of the unknown constant is proposed in this paper. The methodology is based upon the assumption that the R-curve is the same for both flat and curved panels. Experimental data from unstiffened flat and curved panel tests were used to demonstrate the application of this approach. Different values for the empirical constant are calculated for each data point where stable crack extension is measured at a given level of applied load. The average of all these values is taken as the numerical value of the empirical constant.

Failure predictions of two unstiffened curved panels were made using the measured R-curve data and the nonlinear bulging factor as determined by the present technique. Analytical predictions of panel failure were found to be within 10% of the experimental values.

**Acknowledgement** - The research described in this paper was performed in the support of the Federal Aviation Administration Technical Center's Aging Aircraft Research Program. The support and interest from Dr. Michael L. Basehore at FAATC is gratefully appreciated.

## REFERENCES

- [1] Kuhn, P.: Notch Effects on Fatigue and Static Strength, Symposium of the International Committee on Aeronautical Fatigue (ICAF), Rome, Italy, 1963.
- [2] Folias, E.S.: An Axial Crack in a Pressurized Cylindrical Shell, *International Journal of Fracture Mechanics* 1, 104-113, 1965.
- [3] Folias, E.S.: On the Prediction of Catastrophic Failures in Pressurized Vessels, *Perspectives of Fracture*, Noordhoff International, Leyden, The Netherlands, 405-418, 1974.
- [4] Erdogan, F.; and Kibler, J.J.: Cylindrical and Spherical Shells with Cracks, *International Journal of Fracture Mechanics* 5, 229-236, 1969.
- [5] Ansell, H.: Bulging of Cracked Pressurized Aircraft Structure, Ph.D. Thesis, Linköping Institute of Technology, Sweden, 1988.
- [6] Chen, D.: Bulging of Fatigue Cracks in a Pressurized Aircraft Fuselage, Ph.D. Thesis, Delft University of Technology, 1991.
- [7] Broek, D.: The Effects of Multi-Site Damage on the Arrest Capability of Aircraft Fuselage Structures, Fracture Research Report TR 9302, June 1993.
- [8] Timoshenko, S.; and Woinowsky-Krieger, S.: *Theory of Plates and Shells*, Second Edition, McGraw-Hill, New York, NY, 1959.
- [9] Thomson, D.; Hoadley, D.; and McHatton, J.; Load Tests of Flat and Curved Panels with Multiple Cracks, Foster-Miller, Inc. Draft Final Report to Volpe Center, September 1993.